Instability of rotating chiral solitons

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We show that spherically symmetric chiral SU(2)×SU(2) solitons are unstable under spin-isospin rotations. Namely, the effective potential including the effects of quantizing the collective coordinate corresponding to such a rotation has no minimum in the class of functions used to describe such solitons.

The idea that baryons may emerge as solitons[1] in a $U(N)\times U(N)$ chiral model has been recently given a new impetus.[2] The phenomenology of static nucleons and nucleon resonances has been studied.[3] A mixed picture,[4] wherein for small distances the nucleon is viewed as an MIT bag consisting of three quarks and for large distances as a chiral soliton, has also been studied. Of course, a complete treatment has to include a quantization of all modes around the soliton solution. A partial quantization was done in Ref. 3. In that work, the mode corresponding to a joint spin-isospin rotation, which is a constant of the motion for the soliton, was quantized. The procedure followed was first to solve the static equations for the soliton and then to rotate this *fixed* solution in spin and isospin coordinates. In one-particle quantum mechanics, an analogous procedure would consist of finding the minimum, $r=r_0$, of a potential $V(r)$ and then adding a rotational energy with fixed moment of inertia, $I=mr_0^2$. To this order, the energy of a state with angular momentum l would be $V(r_0)+l(l+1)/2mr_0^2$. A somewhat more accurate procedure would be first to diagonalize the angular momentum, obtaining an effective potential $V_{\text{eff}}(r)=V(r)+l(l+1)/2mr^2$, and then to minimize $V_{\text{eff}}(r)$. Had the minimum of $V(r)$ been very steep, the difference in these two procedures would not have been, for small l, very significant. For large values of l, $V_{\text{eff}}(r)$ will, in general, cease to have a minimum. In this note we will show that if we follow the second procedure, then in the chiral model there will be *no* spherical soliton solution for any value of the spin-isospin.

The Skyrme model[1] is based on the Lagrangian

$$L=\frac{F_\pi^2}{16}\mathrm{Tr}[\partial_\mu U\partial_\mu U^\dagger]+\cdots\ . \tag{1}$$

U is an SU(2) matrix and F_π is related to the pion decay constant, $F_\pi=186$ MeV. The dots refer to terms quartic (and possibly of higher order) in U. The latter are essential in order to prevent the soliton from shrinking to zero size. As we will be concerned with large-distance behavior, their details will not be important. For the sake of notational compactness we will denote all contributions arising from such terms by dots.

The static potential energy corresponding to (1) is

$$V[U]=\frac{F_\pi^2}{16}\int d^3r\,\mathrm{Tr}[\vec\nabla U\cdot\vec\nabla U^\dagger]+\cdots\ . \tag{2}$$

The Skyrme soliton is a minimum of this potential in a class of functions $F(r)$, with $F(0)=\pi$ and $F(\infty)=0$, based on the ansatz

$$U(\vec r)=\exp[iF(r)\vec\tau\cdot\hat r]\ , \tag{3}$$

where $\hat r$ is a unit vector in the r direction, $\vec\tau$ the Pauli matrices. Spin-isospin rotations correspond to the transformation

$$U\rightarrow AUA^\dagger\ , \tag{4}$$

with A a space-independent SU(2) matrix. Diagonalizing the Hamiltonian for the motion corresponding to A we obtain an effective potential[3]

$$\begin{aligned}V_{\text{eff}}[F]&=M[F]+l(l+2)/8\lambda[F]\ ,\\ M[F]&=\frac{F_\pi^2}{8}4\pi\int r^2dr\left[\left(\frac{\partial F}{\partial r}\right)^2+\frac{2\sin^2F}{r^2}\right]+\cdots\ ,\\ \lambda[F]&=\frac{F_\pi^2}{6}4\pi\int r^2dr\sin^2F+\cdots\ .\end{aligned} \tag{5}$$

In the above $l=2J=2I$, where J and I are the spin and isospin of the nucleon states. Minimizing $M[F]$ yields the usual soliton solutions. We shall show that, for any l, $V_{\text{eff}}[F]$ has no minimum within the class of functions considered.

The Euler-Lagrange equation for the minimum of V_{eff} is

$$\pi F_\pi^2\left(-\frac{\partial}{\partial r}r^2\frac{\partial F}{\partial r}+\sin 2F+\cdots\right)-\frac{3l(l+2)}{16\pi F_\pi^2}\left(\frac{r^2\sin 2F+\cdots}{\left(\int r^2dr\sin^2F+\cdots\right)^2}\right)=0\ . \tag{6}$$

We claim that for $l\neq 0$ these equations do not have a solution. For large r we seek solutions with $F\rightarrow 0$ and thus we may linearize Eq. (6),

$$-\frac{\partial^2F}{\partial r^2}-\frac{2}{r}\frac{\partial F}{\partial r}+\frac{2}{r^2}F-\frac{3}{8}\frac{l(l+2)}{\pi^2F_\pi^4}\frac{F}{\left(\int r^2dr\sin^2F+\cdots\right)^2}=0\ . \tag{7}$$

[The quartic terms in the Lagrangian contribute only to the integral in the denominator of Eq. (7). Other terms decrease faster then the ones retained.] Equation (7) has the form of a free-particle radial Schrödinger equation with angular momentum equal to one and wave number

$$k^2=\frac{3l(l+2)}{8\pi^2F_\pi^4\left(\int r^2dr\sin^2F+\cdots\right)^2}\ . \tag{8}$$

Asymptotically F behaves as $(\sin kr)/r$ or $(\cos kr)/r$ and the integral in the denominator of Eq. (8) does not converge.

For $k^2=0$, F goes as $1/r^2$ yielding a convergent value for the integral and a nonzero value for k^2. This contradiction shows that Eq. (6) has no solutions for any finite value of $l/F_\pi{}^2$.

If one treats the chiral, low-energy, Lagrangian as an effective Lagrangian for QCD in the large-N limit (N is the number of colors), then the rotational term in the Lagrangian is formally of order $1/N^2$ relative to the main term.[3] Quantization of the rotational degrees of freedom can then be considered as semiclassical with $1/N$ being the small parameter. The relative $1/N^2$ factor between the static and rotational terms obtained in Ref. 3 is due to the fact that the characteristic distances in $F(r)$ are $rF_\pi \sim \sqrt{N}$. When $V_{\rm eff}$ is minimized, much larger distances become relevant changing the N dependence of these terms.

The breaking of chiral symmetry through the addition of a pion mass[5] modifies Eq. (7) by adding a term proportional to $m_\pi{}^2 F$ to it. We now expect, at least for small l, exponentially damped solutions of Eq. (6). Of course, all nucleon properties would not be smooth in the limit $m_\pi \rightarrow 0$, contradicting the hypothesis that at least nucleon masses have a smooth chiral limit.

A different regularization of this problem is achieved by introducing a radius R, with $F(R)=0$, and at the end letting $R \rightarrow \infty$. For finite R, we solve Eq. (6) with k^2 of Eq. (8) fixed and then use this equation to determine it self-consistently. For large R, $k \sim 1/R$ and an analysis of Eq. (6) shows that the rotations do not contribute to the energy.

Topics remaining to be investigated are whether deformed (nonspherical) solitons are stable under rotations and/or whether additional time dependences will dampen the large-distance oscillation. Although the contributions of these additional vibrations is expected to be still of higher order in $1/N$, it is possible for them to regulate the rotational instability and in fact restore the results of Ref. 3. Should any of these approaches prove correct, then the centrifugal barrier introduced by rotations would stabilize the soliton against collapse to smaller sizes, removing the necessity for the quartic term in the Lagrangian.

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